Decomposable Graphs are Set Recognizable

Bernd Schröder

Definition.

Definition. *Let* $G = (V, E)$ *be a graph.*

Definition. *Let $G = (V, E)$ be a graph. The set $\{G - v : v \in V\}$ of unlabeled one-vertex-deleted subgraphs is called the* **set deck** *of G.*

Definition. *Let $G = (V, E)$ be a graph. The set $\{G - v : v \in V\}$ of unlabeled one-vertex-deleted subgraphs is called the* **set deck** *of G.*

The set reconstruction problem asks if two graphs with the same set deck (and ≥ 3 vertices, which will not be mentioned ad infinitum) must be isomorphic.

Definition. *Let $G = (V, E)$ be a graph. The set $\{G - v : v \in V\}$ of unlabeled one-vertex-deleted subgraphs is called the* **set deck** *of G.*

The set reconstruction problem asks if two graphs with the same set deck (and ≥ 3 vertices, which will not be mentioned ad infinitum) must be isomorphic.

Disconnected graphs are set reconstructible.

Definition. *Let $G=(V,E)$ be a graph. The set $\{G-v : v \in V\}$ of unlabeled one-vertex-deleted subgraphs is called the* **set deck** *of G.*

The set reconstruction problem asks if two graphs with the same set deck (and ≥ 3 vertices, which will not be mentioned ad infinitum) must be isomorphic.

Disconnected graphs are set reconstructible.

Set reconstructibility transfers to the complement.

Definition.

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable**

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

These "... unless" are what makes (set) reconstruction hard

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

These “... unless” are what makes (set) reconstruction hard, the “... in which case” are what kindles hope for a solution

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

These “... unless” are what makes (set) reconstruction hard, the “... in which case” are what kindles hope for a solution, but they also make the search tedious.

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

These "... unless" are what makes (set) reconstruction hard, the "... in which case" are what kindles hope for a solution, but they also make the search tedious.

Properties, like decomposability, can of course be encoded as a parameter

Definition. *A parameter (like the size of the largest clique) is called* **set recognizable** *iff the parameter must be the same for any two graphs with equal set decks.*

For the size of the largest clique, find the largest clique on any card, that is the number we are looking for ... unless all cards are complete graphs, in which case the set deck consists of 1 card, which is a complete graph, so you add 1.

These "... unless" are what makes (set) reconstruction hard, the "... in which case" are what kindles hope for a solution, but they also make the search tedious.

Properties, like decomposability, can of course be encoded as a parameter: Set it to 1 when the graph is decomposable, set it to 0 when not.

Definition.

Definition. *Let* $G = (V, E)$ *be a graph and let* $A \subseteq V$*.*

Definition. *Let* $G = (V, E)$ *be a graph and let* $A \subseteq V$*. Then* A *is called* **autonomous**

Definition. *Let* $G = (V, E)$ *be a graph and let* $A \subseteq V$. *Then* A *is called* **autonomous** *iff, for all* $v \in V \setminus A$, *we have that*

Definition. *Let $G = (V, E)$ be a graph and let $A \subseteq V$. Then A is called* **autonomous** *iff, for all $v \in V \setminus A$, we have that*

- *If there is an $a \in A$ with $v \sim a$, then, for all $z \in A$, we have $v \sim z$.*

Definition. *Let $G = (V, E)$ be a graph and let $A \subseteq V$. Then A is called* **autonomous** *iff, for all $v \in V \setminus A$, we have that*

- *If there is an $a \in A$ with $v \sim a$, then, for all $z \in A$, we have $v \sim z$.*

An autonomous subset A such that $|A| \notin \{0, 1, |V|\}$ is called **nontrivial**.

Definition. *Let $G=(V,E)$ be a graph and let $A\subseteq V$. Then A is called* **autonomous** *iff, for all $v\in V\setminus A$, we have that*

- *If there is an $a\in A$ with $v\sim a$, then, for all $z\in A$, we have $v\sim z$.*

An autonomous subset A such that $|A|\notin\{0,1,|V|\}$ is called **nontrivial**. *A graph with a nontrivial order-autonomous set of vertices A is called* **decomposable**.

Definition. *Let $G = (V, E)$ be a graph and let $A \subseteq V$. Then A is called* **autonomous** *iff, for all $v \in V \setminus A$, we have that*

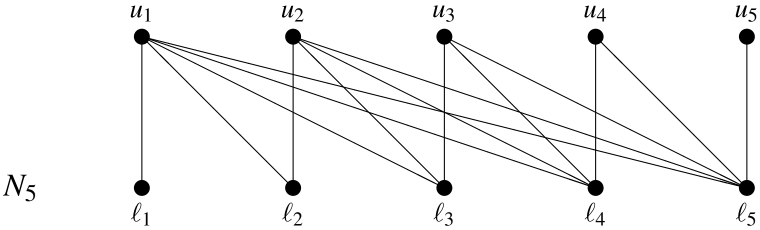
- *If there is an $a \in A$ with $v \sim a$, then, for all $z \in A$, we have $v \sim z$.*

An autonomous subset A such that $|A| \notin \{0, 1, |V|\}$ is called **nontrivial**. *A graph with a nontrivial order-autonomous set of vertices A is called* **decomposable**.

Autonomy and decomposability transfer to the complement.

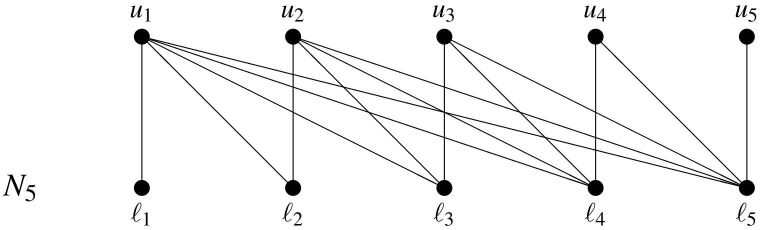
An Indecomposable Graph

An Indecomposable Graph

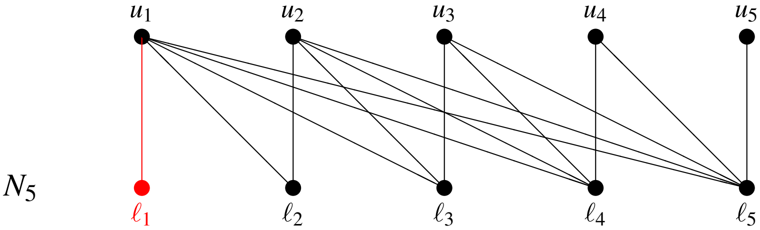


Decomposable Graphs

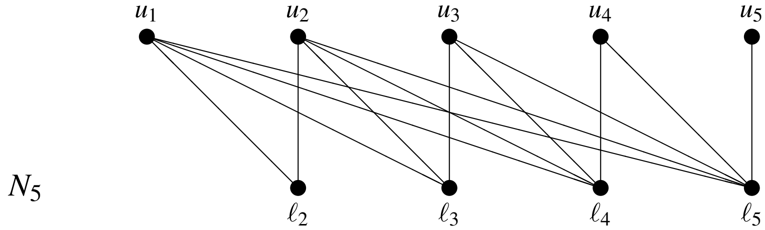
Decomposable Graphs



Decomposable Graphs

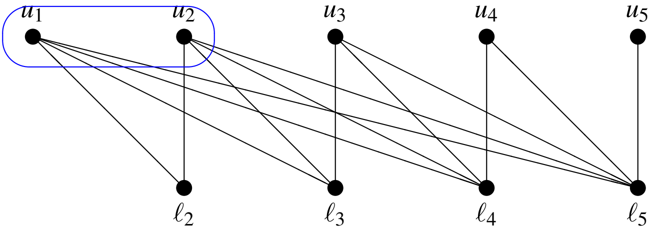


Decomposable Graphs

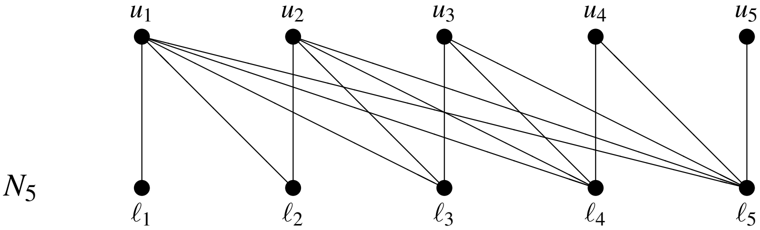


Decomposable Graphs

N_5

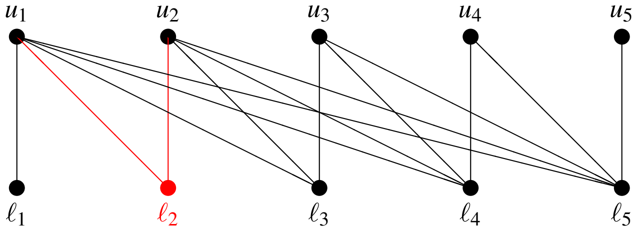


Decomposable Graphs



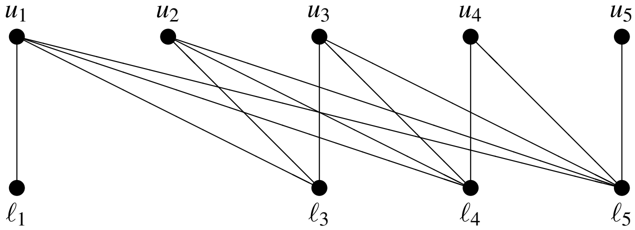
Decomposable Graphs

N_5

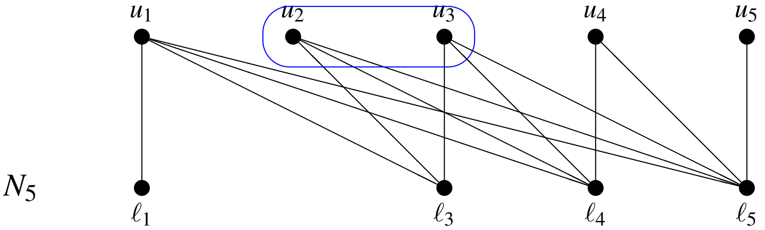


Decomposable Graphs

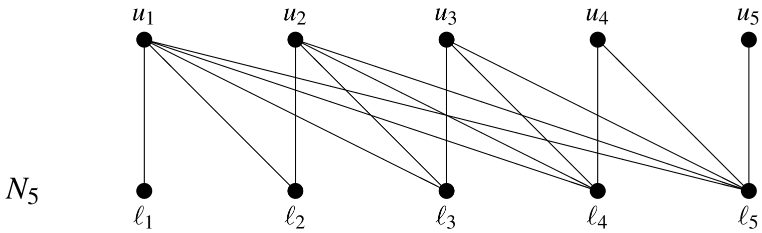
N_5



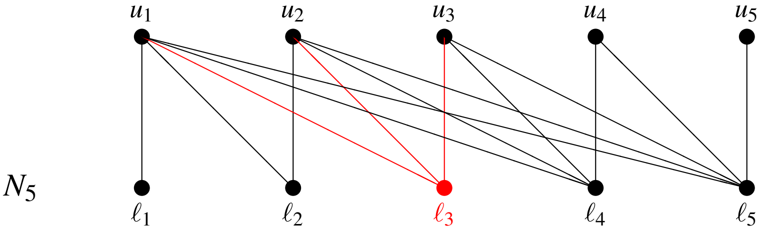
Decomposable Graphs



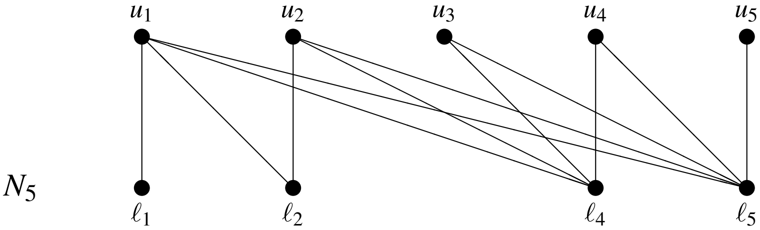
Decomposable Graphs



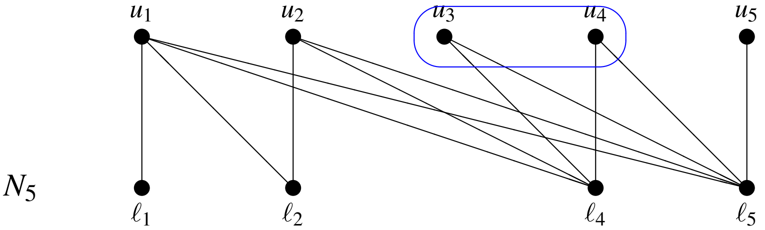
Decomposable Graphs



Decomposable Graphs

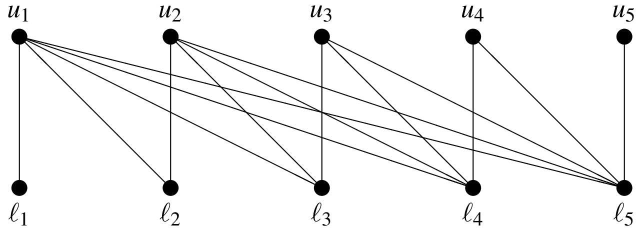


Decomposable Graphs

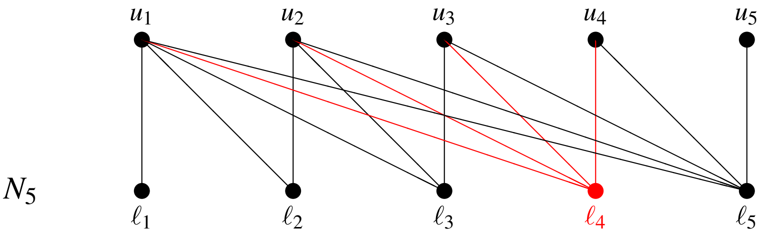


Decomposable Graphs

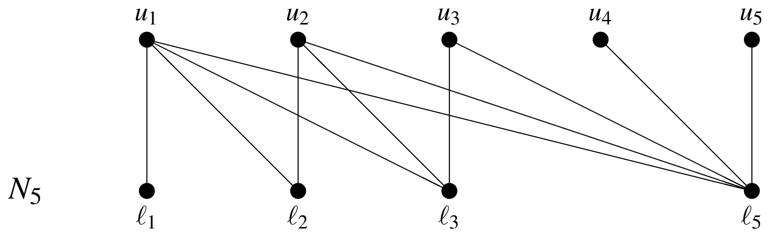
N_5



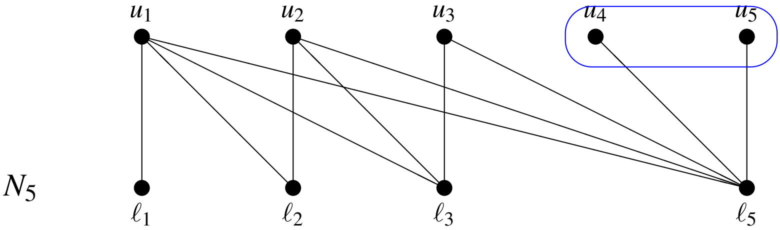
Decomposable Graphs



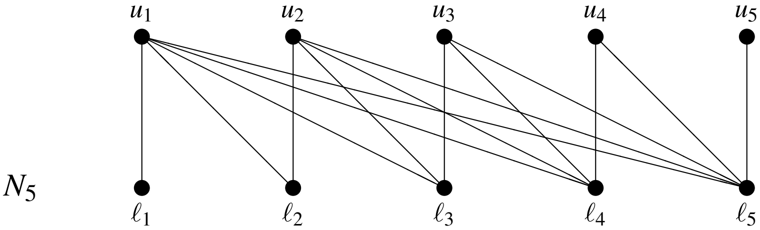
Decomposable Graphs



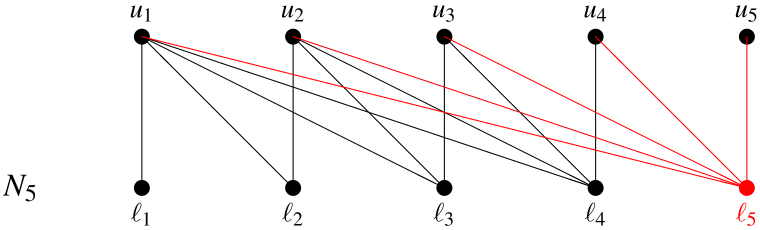
Decomposable Graphs



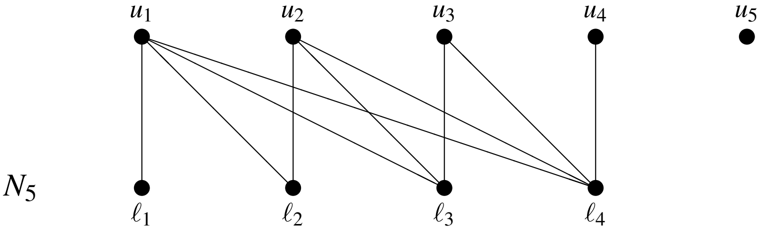
Decomposable Graphs



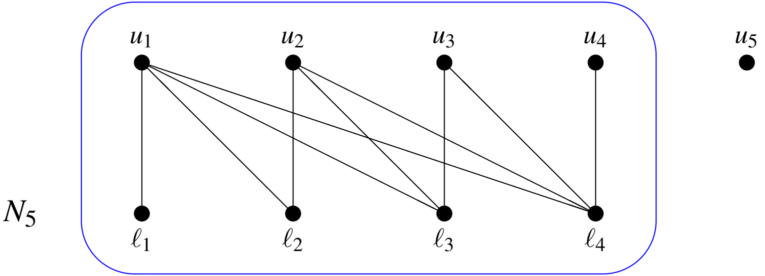
Decomposable Graphs



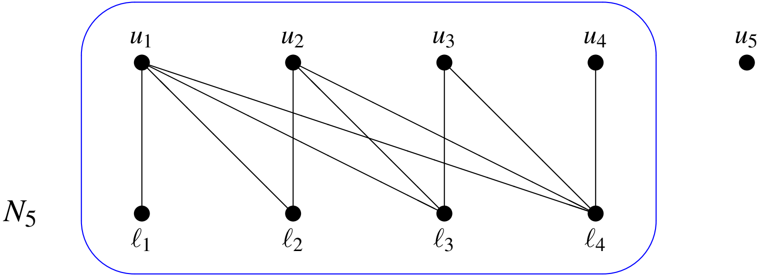
Decomposable Graphs



Decomposable Graphs

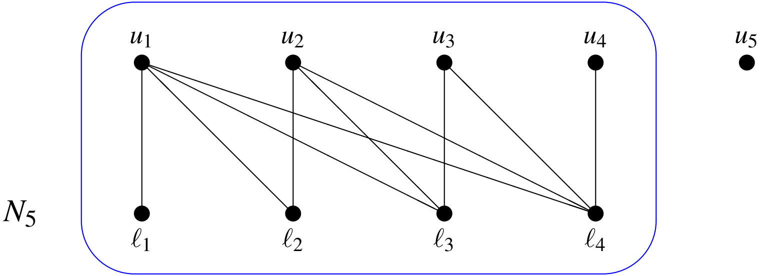


Decomposable Graphs



I would have loved to prove that these and their complements were the only indecomposable graphs such that all cards are decomposable

Decomposable Graphs



I would have loved to prove that these and their complements were the only indecomposable graphs such that all cards are decomposable, but Schmerl and Trotter did that in 1993.

Cards of Decomposable Graphs

Cards of Decomposable Graphs

Let $G = (V, E)$ be decomposable.

Cards of Decomposable Graphs

Let $G = (V, E)$ be decomposable.

- Then every card of G is decomposable

Cards of Decomposable Graphs

Let $G = (V, E)$ be decomposable.

- Then every card of G is decomposable ...
- ... unless G has exactly one nontrivial autonomous set of vertices and that set has exactly 2 vertices.

Cards of Decomposable Graphs

Let $G = (V, E)$ be decomposable.

- Then every card of G is decomposable ...
- ... unless G has exactly one nontrivial autonomous set of vertices and that set has exactly 2 vertices.
- In that case, there are two isomorphic indecomposable cards

Cards of Decomposable Graphs

Let $G = (V, E)$ be decomposable.

- Then every card of G is decomposable ...
- ... unless G has exactly one nontrivial autonomous set of vertices and that set has exactly 2 vertices.
- In that case, there are two isomorphic indecomposable cards and all other cards have an autonomous set of vertices with 2 cards.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

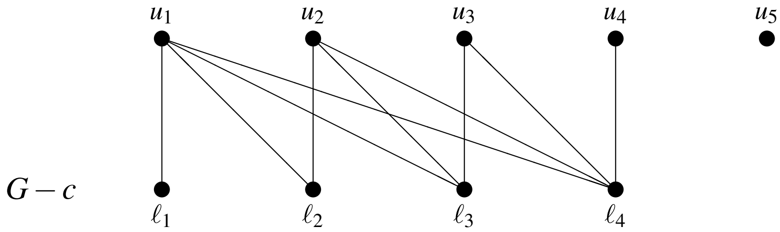
Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

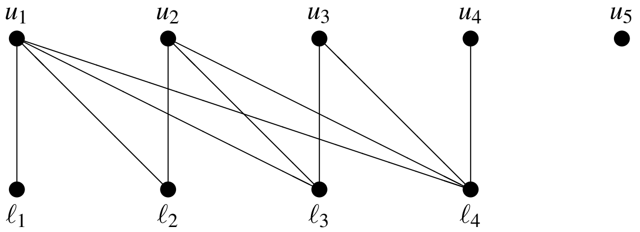
Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.



Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.

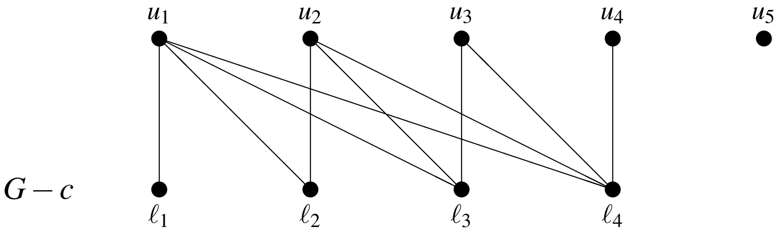
$G - c$



- Then $c \notin B$.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

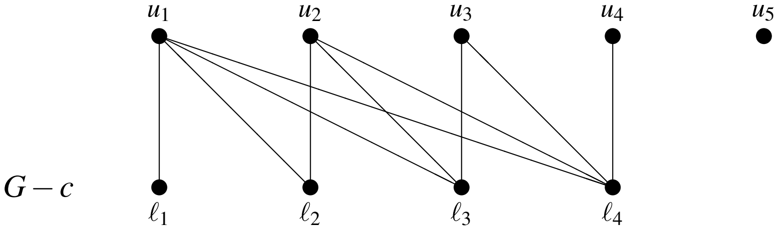
Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.



- Then $c \notin B$.
- Then c is adjacent to all vertices in $V \setminus \{c\}$.

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

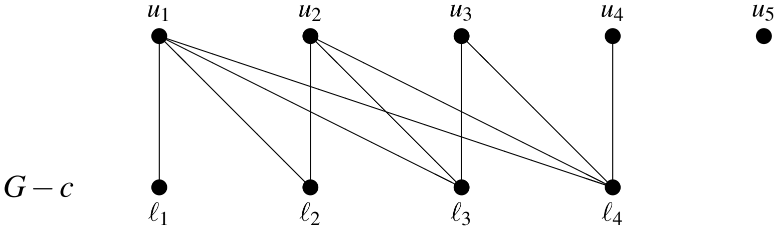
Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.



- Then $c \notin B$.
- Then c is adjacent to all vertices in $V \setminus \{c\}$.
- Then G^c is disconnected

Indecomposable Graphs with All Cards Decomposable are Set Reconstructible

Suppose, for a contradiction, G was decomposable and had the same set deck as a graph N_k. Then G and G^c are connected. Let B be nontrivially autonomous.



- Then $c \notin B$.
- Then c is adjacent to all vertices in $V \setminus \{c\}$.
- Then G^c is disconnected, contradiction.

Definition.

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff*

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff the indecomposable cards of* G *are pairwise isomorphic*

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff the indecomposable cards of* G *are pairwise isomorphic, and one of the following holds.*

Definition. *A graph $G = (V, E)$ is called* **almost-all-cards-decomposable** *iff the indecomposable cards of G are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of G contains an autonomous set of vertices A that consists of 2 independent vertices. In this case, G is called an* **edge-absent almost-all-cards-decomposable** *graph.*

Definition. *A graph $G = (V, E)$ is called* **almost-all-cards-decomposable** *iff the indecomposable cards of G are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of G contains an autonomous set of vertices A that consists of 2 independent vertices. In this case, G is called an* **edge-absent almost-all-cards-decomposable** *graph. Each such set A is called a* **preferred nonedge***.*

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff the indecomposable cards of G are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of G contains an autonomous set of vertices A that consists of 2 independent vertices. In this case, G is called an* **edge-absent almost-all-cards-decomposable** *graph. Each such set A is called a* **preferred nonedge**.
2. *Every decomposable card of G contains an edge A that is an autonomous set of vertices.*

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff the indecomposable cards of* G *are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of* G *contains an autonomous set of vertices* A *that consists of* 2 *independent vertices. In this case,* G *is called an* **edge-absent almost-all-cards-decomposable** *graph. Each such set* A *is called a* **preferred nonedge**.
2. *Every decomposable card of* G *contains an edge* A *that is an autonomous set of vertices. In this case,* G *is called an* **edge-present almost-all-cards-decomposable** *graph.*

Definition. *A graph $G = (V, E)$ is called* **almost-all-cards-decomposable** *iff the indecomposable cards of G are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of G contains an autonomous set of vertices A that consists of 2 independent vertices. In this case, G is called an* **edge-absent almost-all-cards-decomposable** *graph. Each such set A is called a* **preferred nonedge**.
2. *Every decomposable card of G contains an edge A that is an autonomous set of vertices. In this case, G is called an* **edge-present almost-all-cards-decomposable** *graph. Each such set A is called a* **preferred edge**.

Definition. *A graph* $G = (V, E)$ *is called* **almost-all-cards-decomposable** *iff the indecomposable cards of* G *are pairwise isomorphic, and one of the following holds.*

1. *Every decomposable card of* G *contains an autonomous set of vertices* A *that consists of* 2 *independent vertices. In this case,* G *is called an* **edge-absent almost-all-cards-decomposable** *graph. Each such set* A *is called a* **preferred nonedge**.
2. *Every decomposable card of* G *contains an edge* A *that is an autonomous set of vertices. In this case,* G *is called an* **edge-present almost-all-cards-decomposable** *graph. Each such set* A *is called a* **preferred edge**.

Decomposable with exactly one autonomous set which is a doubleton implies almost-all-cards-decomposable.

Definition.

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable*

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G.*

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Definition. *Let $G=(V,E)$ be a graph, let $z\in V$ be so that $G-z$ is decomposable and let $A\subset V$ be a nontrivial autonomous set of vertices of $G-z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma.

Definition. *Let $G=(V,E)$ be a graph, let $z\in V$ be so that $G-z$ is decomposable and let $A\subset V$ be a nontrivial autonomous set of vertices of $G-z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G=(V,E)$ be a graph such that $z\in V$ binds $A\subseteq V$.*

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G = (V, E)$ be a graph such that $z \in V$ binds $A \subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G = (V, E)$ be a graph such that $z \in V$ binds $A \subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G = (V, E)$ be a graph such that $z \in V$ binds $A \subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Proof.

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G = (V, E)$ be a graph such that $z \in V$ binds $A \subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Proof. $z \sim A$ or z not adjacent to any vertex in A

Definition. *Let $G = (V, E)$ be a graph, let $z \in V$ be so that $G - z$ is decomposable and let $A \subset V$ be a nontrivial autonomous set of vertices of $G - z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G = (V, E)$ be a graph such that $z \in V$ binds $A \subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Proof. $z \sim A$ or z not adjacent to any vertex in A would make A autonomous in G.

Definition. *Let $G=(V,E)$ be a graph, let $z\in V$ be so that $G-z$ is decomposable and let $A\subset V$ be a nontrivial autonomous set of vertices of $G-z$ that is not autonomous in G. Then we say that z* **binds** *A.*

Lemma. *Let $G=(V,E)$ be a graph such that $z\in V$ binds $A\subseteq V$. Then z is adjacent to some, but not all, vertices of A.*

Proof. $z\sim A$ or z not adjacent to any vertex in A would make A autonomous in G. ■

Lemma.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z.*

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph and let* A *be a preferred nonedge that is bound by the vertex* z*. Then there is a* $z' \in A$ *which binds a preferred nonedge* A'*.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof. Let $A = \{z', a\}$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof. Let $A = \{z', a\}$. Because $G - z'$ and $G - a$ are not isomorphic, one of them is decomposable.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof. Let $A = \{z', a\}$. Because $G - z'$ and $G - a$ are not isomorphic, one of them is decomposable. Without loss of generality, let $G - z'$ be decomposable.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof. Let $A=\{z',a\}$. Because $G-z'$ and $G-a$ are not isomorphic, one of them is decomposable. Without loss of generality, let $G-z'$ be decomposable. Then z' binds a preferred nonedge.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph and let A be a preferred nonedge that is bound by the vertex z. Then there is a $z' \in A$ which binds a preferred nonedge A'.*

Proof. Let $A = \{z', a\}$. Because $G - z'$ and $G - a$ are not isomorphic, one of them is decomposable. Without loss of generality, let $G - z'$ be decomposable. Then z' binds a preferred nonedge. ∎

Lemma.

Lemma. *Let $G = (V, E)$ be an indecomposable graph.*

Lemma. *Let $G=(V,E)$ be an indecomposable graph. Let $A,A'\subseteq V$ be bound sets of vertices*

Lemma. *Let* $G = (V, E)$ *be an indecomposable graph. Let* $A, A' \subseteq V$ *be bound sets of vertices, bound by* $z, z' \in V$*, respectively*

Lemma. *Let* $G = (V, E)$ *be an indecomposable graph. Let* $A, A' \subseteq V$ *be bound sets of vertices, bound by* $z, z' \in V$*, respectively, and such that* $z' \in A$

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$.*

Lemma. *Let* $G = (V, E)$ *be an indecomposable graph. Let* $A, A' \subseteq V$ *be bound sets of vertices, bound by* $z, z' \in V$*, respectively, and such that* $z' \in A$ *and* $z \notin A'$*. Then* $A \cap A' \neq \emptyset$*.*

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof.

Lemma. *Let $G = (V,E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$.

Lemma. *Let $G = (V,E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'.

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A \setminus \{z'\} \neq \emptyset$.

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A \setminus \{z'\} \neq \emptyset$. Let $a \in A \setminus \{z'\}$.

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A \setminus \{z'\} \neq \emptyset$. Let $a \in A \setminus \{z'\}$. Because $A' \cap A = \emptyset$, we have $a \in V \setminus (A' \cup \{z'\})$.

Lemma. *Let $G = (V, E)$ be an indecomposable graph. Let $A, A' \subseteq V$ be bound sets of vertices, bound by $z, z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A \setminus \{z'\} \neq \emptyset$. Let $a \in A \setminus \{z'\}$. Because $A' \cap A = \emptyset$, we have $a \in V \setminus (A' \cup \{z'\})$. Because A is an autonomous set of vertices in $G - z$, because $z' \in A$ and because $z \notin A'$, we conclude that a, like z', is adjacent to some, but not all, vertices in A'

Lemma. *Let $G=(V,E)$ be an indecomposable graph. Let $A,A'\subseteq V$ be bound sets of vertices, bound by $z,z'\in V$, respectively, and such that $z'\in A$ and $z\notin A'$. Then $A\cap A'\neq\emptyset$.*

Proof. Suppose, for a contradiction, that $A'\cap A=\emptyset$. $z'\in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A\setminus\{z'\}\neq\emptyset$. Let $a\in A\setminus\{z'\}$. Because $A'\cap A=\emptyset$, we have $a\in V\setminus(A'\cup\{z'\})$. Because A is an autonomous set of vertices in $G-z$, because $z'\in A$ and because $z\notin A'$, we conclude that a, like z', is adjacent to some, but not all, vertices in A', a contradiction.

Lemma. *Let $G=(V,E)$ be an indecomposable graph. Let $A,A' \subseteq V$ be bound sets of vertices, bound by $z,z' \in V$, respectively, and such that $z' \in A$ and $z \notin A'$. Then $A \cap A' \neq \emptyset$.*

Proof. Suppose, for a contradiction, that $A' \cap A = \emptyset$. $z' \in A$ is adjacent to some, but not all, vertices in A'. Because A is nontrivial, $A \setminus \{z'\} \neq \emptyset$. Let $a \in A \setminus \{z'\}$. Because $A' \cap A = \emptyset$, we have $a \in V \setminus (A' \cup \{z'\})$. Because A is an autonomous set of vertices in $G-z$, because $z' \in A$ and because $z \notin A'$, we conclude that a, like z', is adjacent to some, but not all, vertices in A', a contradiction. ■

Lemma.

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph.*

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph. Let* $A, A' \subseteq V$ *be preferred nonedges, bound by* $z, z' \in V$*, respectively, and such that* $z' \in A$.

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph. Let* $A, A' \subseteq V$ *be preferred nonedges, bound by* $z, z' \in V$*, respectively, and such that* $z' \in A$*. Then* $z \in A'$

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph. Let* $A, A' \subseteq V$ *be preferred nonedges, bound by* $z, z' \in V$*, respectively, and such that* $z' \in A$*. Then* $z \in A'$ *and* $A \cap A' = \emptyset$*.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A,A' \subseteq V$ be preferred nonedges, bound by $z,z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A=\{z',a\}$.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.
Because $|A'| = 2$, there is a unique vertex $a' \in A' \setminus A$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.
Because $|A'| = 2$, there is a unique vertex $a' \in A' \setminus A$.
Because $A = \{z', a\}$ is an independent set of vertices, we have $z' \not\sim a$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.
Because $|A'| = 2$, there is a unique vertex $a' \in A' \setminus A$.
Because $A = \{z', a\}$ is an independent set of vertices, we have $z' \not\sim a$. Because z' binds $A' = \{a, a'\}$, we infer that $z' \sim a'$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.
Because $|A'| = 2$, there is a unique vertex $a' \in A' \setminus A$.
Because $A = \{z', a\}$ is an independent set of vertices, we have $z' \not\sim a$. Because z' binds $A' = \{a, a'\}$, we infer that $z' \sim a'$.
Because $a' \in V \setminus (A \cup \{z\})$ and because A is an autonomous set of vertices in $G - z$, the adjacency $a' \sim z'$ implies $a' \sim a$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A,A'\subseteq V$ be preferred nonedges, bound by $z,z'\in V$, respectively, and such that $z'\in A$. Then $z\in A'$ and $A\cap A'=\emptyset$.*

Proof. Let $A=\{z',a\}$.
Suppose, for a contradiction that $z\notin A'$. Then $A\cap A'\neq\emptyset$.
Because $A=\{z',a\}$ and $z'\notin A'$, we obtain $A\cap A'=\{a\}$.
Because $|A'|=2$, there is a unique vertex $a'\in A'\setminus A$.
Because $A=\{z',a\}$ is an independent set of vertices, we have $z'\not\sim a$. Because z' binds $A'=\{a,a'\}$, we infer that $z'\sim a'$.
Because $a'\in V\setminus(A\cup\{z\})$ and because A is an autonomous set of vertices in $G-z$, the adjacency $a'\sim z'$ implies $a'\sim a$. This is a contradiction to A' being a preferred nonedge.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof. Let $A = \{z', a\}$.
Suppose, for a contradiction that $z \notin A'$. Then $A \cap A' \neq \emptyset$.
Because $A = \{z', a\}$ and $z' \notin A'$, we obtain $A \cap A' = \{a\}$.
Because $|A'| = 2$, there is a unique vertex $a' \in A' \setminus A$.
Because $A = \{z', a\}$ is an independent set of vertices, we have $z' \not\sim a$. Because z' binds $A' = \{a, a'\}$, we infer that $z' \sim a'$.
Because $a' \in V \setminus (A \cup \{z\})$ and because A is an autonomous set of vertices in $G - z$, the adjacency $a' \sim z'$ implies $a' \sim a$. This is a contradiction to A' being a preferred nonedge. Hence we must have $z \in A'$.

Lemma. *Let* $G=(V,E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph. Let* $A,A'\subseteq V$ *be preferred nonedges, bound by* $z,z'\in V$*, respectively, and such that* $z'\in A$*. Then* $z\in A'$ *and* $A\cap A'=\emptyset$*.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.).

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A,A'\subseteq V$ be preferred nonedges, bound by $z,z'\in V$, respectively, and such that $z'\in A$. Then $z\in A'$ and $A\cap A'=\emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A'\cap A\neq\emptyset$. Claim: $A'\cup A$ is an autonomous set of vertices in G.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$. In particular, v is adjacent to a vertex in A'.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$. In particular, v is adjacent to a vertex in A'. Now, because $v \in V \setminus (A' \cup \{z'\})$, we have $v \sim A'$.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$. In particular, v is adjacent to a vertex in A'. Now, because $v \in V \setminus (A' \cup \{z'\})$, we have $v \sim A'$. Thus $v \sim A' \cup A$

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$. In particular, v is adjacent to a vertex in A'. Now, because $v \in V \setminus (A' \cup \{z'\})$, we have $v \sim A'$. Thus $v \sim A' \cup A$ and $A' \cup A$ is an autonomous set of vertices in G.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A,A'\subseteq V$ be preferred nonedges, bound by $z,z'\in V$, respectively, and such that $z'\in A$. Then $z\in A'$ and $A\cap A'=\emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A'\cap A\neq\emptyset$. Claim: $A'\cup A$ is an autonomous set of vertices in G. Let $v\in V\setminus(A'\cup A)$ be so that there is an $x\in A'\cup A$ with $v\sim x$. Without loss of generality, let $x\in A$. Because $v\in V\setminus(A\cup\{z\})$, we have $v\sim A$. In particular, v is adjacent to a vertex in A'. Now, because $v\in V\setminus(A'\cup\{z'\})$, we have $v\sim A'$. Thus $v\sim A'\cup A$ and $A'\cup A$ is an autonomous set of vertices in G. Contradiction.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Let $A, A' \subseteq V$ be preferred nonedges, bound by $z, z' \in V$, respectively, and such that $z' \in A$. Then $z \in A'$ and $A \cap A' = \emptyset$.*

Proof (cont.). Suppose, for a contradiction, that $A' \cap A \neq \emptyset$. Claim: $A' \cup A$ is an autonomous set of vertices in G. Let $v \in V \setminus (A' \cup A)$ be so that there is an $x \in A' \cup A$ with $v \sim x$. Without loss of generality, let $x \in A$. Because $v \in V \setminus (A \cup \{z\})$, we have $v \sim A$. In particular, v is adjacent to a vertex in A'. Now, because $v \in V \setminus (A' \cup \{z'\})$, we have $v \sim A'$. Thus $v \sim A' \cup A$ and $A' \cup A$ is an autonomous set of vertices in G. Contradiction. ■

Lemma.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x, z\})$, then

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x, z\})$, then, because A and B are autonomous sets of vertices in $G - z$

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x, z\})$, then, because A and B are autonomous sets of vertices in $G - z$, we obtain first $v \sim x$

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x,z\})$, then, because A and B are autonomous sets of vertices in $G-z$, we obtain first $v \sim x$ and then $v \sim (A \cup B) \setminus \{x\} = C$.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x,z\})$, then, because A and B are autonomous sets of vertices in $G - z$, we obtain first $v \sim x$ and then $v \sim (A \cup B) \setminus \{x\} = C$. If $v = z$, then

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x, z\})$, then, because A and B are autonomous sets of vertices in $G - z$, we obtain first $v \sim x$ and then $v \sim (A \cup B) \setminus \{x\} = C$. If $v = z$, then $v \sim c$ implies, first that $v \not\sim x$

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x,z\})$, then, because A and B are autonomous sets of vertices in $G - z$, we obtain first $v \sim x$ and then $v \sim (A \cup B) \setminus \{x\} = C$. If $v = z$, then $v \sim c$ implies, first that $v \not\sim x$ and then that $v \sim (A \cup B) \setminus \{x\} = C$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonege.*

Proof. Suppose, for a contradiction, that $z \in V$ binds the distinct preferred nonedges $A \neq B$.
If $A \cap B \neq \emptyset$, then $|A \cap B| = 1$. Let $\{x\} := A \cap B$ and let $C := (A \cup B) \setminus \{x\}$. Then $|C| = 2$. Now let $v \in V \setminus C$ be such that there is a $c \in C$ with $v \sim c$. Because both A and B are preferred nonedges, we have $v \neq x$. If $v \in V \setminus (C \cup \{x, z\})$, then, because A and B are autonomous sets of vertices in $G - z$, we obtain first $v \sim x$ and then $v \sim (A \cup B) \setminus \{x\} = C$. If $v = z$, then $v \sim c$ implies, first that $v \not\sim x$ and then that $v \sim (A \cup B) \setminus \{x\} = C$. Thus $A \cap B \neq \emptyset$ would imply that C is a nontrivial autonomous set of vertices in G, a contradiction.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.).

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$. Because $A' = \{z, a'\}$ we have $z \not\sim a'$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$. Because $A' = \{z, a'\}$ we have $z \not\sim a'$. Because z binds $B = \{a', b\}$, we have $z \sim b$.

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$. Because $A' = \{z, a'\}$ we have $z \not\sim a'$. Because z binds $B = \{a', b\}$, we have $z \sim b$. Because $B = \{b, a'\}$, we have $b \not\sim a'$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$. Because $A' = \{z, a'\}$ we have $z \not\sim a'$. Because z binds $B = \{a', b\}$, we have $z \sim b$. Because $B = \{b, a'\}$, we have $b \not\sim a'$. Now, because $A' = \{z, a'\}$ is an autonomous set of vertices in $G - z'$, we obtain $b \not\sim z$, a contradiction.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then every vertex z in G binds at most one preferred nonedge.*

Proof (cont.). There is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$. Let a' be such that $A' = \{z, a'\}$. If $a' \notin B$, then, $A' \subseteq V \setminus (B \cup \{z'\})$; because A' is an autonomous set of vertices in $G - z'$ and because $z \in A'$ is adjacent to one vertex in B, but not the other, we would conclude that $a' \in V \setminus \{z'\}$ is adjacent to one vertex in B, but not the other, a contradiction. Thus $a' \in B$. Let b be the other vertex of B. Because $A \cap B = \emptyset$, we have $b \neq z'$. Because $A' = \{z, a'\}$ we have $z \not\sim a'$. Because z binds $B = \{a', b\}$, we have $z \sim b$. Because $B = \{b, a'\}$, we have $b \not\sim a'$. Now, because $A' = \{z, a'\}$ is an autonomous set of vertices in $G - z'$, we obtain $b \not\sim z$, a contradiction. ■

Lemma.

Lemma. *Let* $G = (V, E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph.*

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that*

Lemma. *Let* $G=(V,E)$ *be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a* $w\in V$ *such that* $G-w$ *is indecomposable and such that there are two distinct vertices* z *and* z' *that bind preferred nonedges that do not contain* w*.*

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G-w$ is indecomposable and w is contained in two distinct preferred nonedges $B=\{z,w\}$ and $B'=\{z',w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge A'

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B

Lemma. *Let $G = (V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B, and A' does not intersect B'.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B, and A' does not intersect B'.
If there is a $w \in V$ such that $G - w$ is indecomposable and w is not contained in any preferred nonedges

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B, and A' does not intersect B'.
If there is a $w \in V$ such that $G - w$ is indecomposable and w is not contained in any preferred nonedges, then there is a z that induces a preferred nonedge A

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w \in V$ such that $G - w$ is indecomposable and w is contained in two distinct preferred nonedges $B = \{z, w\}$ and $B' = \{z', w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B, and A' does not intersect B'.
If there is a $w \in V$ such that $G - w$ is indecomposable and w is not contained in any preferred nonedges, then there is a z that induces a preferred nonedge A and there is a $z' \in A$ that binds a preferred nonedge $A' \neq A$.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w\in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof. If there is a $w\in V$ such that $G-w$ is indecomposable and w is contained in two distinct preferred nonedges $B=\{z,w\}$ and $B'=\{z',w\}$, then z binds a preferred nonedge A and z' binds a preferred nonedge $A'$and A does not intersect B, and A' does not intersect B'.
If there is a $w\in V$ such that $G-w$ is indecomposable and w is not contained in any preferred nonedges, then there is a z that induces a preferred nonedge A and there is a $z'\in A$ that binds a preferred nonedge $A'\neq A$. By assumption, neither A, nor A', contains w.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.).

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that every $w \in V$ such that $G-w$ is indecomposable is contained in exactly one preferred nonedge.

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w\in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that every $w\in V$ such that $G-w$ is indecomposable is contained in exactly one preferred nonedge.
If there are 3 distinct preferred nonedges A_1,A_2,A_3, bound by z_1,z_2,z_3

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that every $w \in V$ such that $G - w$ is indecomposable is contained in exactly one preferred nonedge.
If there are 3 distinct preferred nonedges A_1, A_2, A_3, bound by z_1, z_2, z_3, because every $w \in V$ such that $G - w$ is indecomposable is contained in at most one of A_1, A_2, A_3

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that every $w \in V$ such that $G - w$ is indecomposable is contained in exactly one preferred nonedge.
If there are 3 distinct preferred nonedges A_1, A_2, A_3, bound by z_1, z_2, z_3, because every $w \in V$ such that $G - w$ is indecomposable is contained in at most one of A_1, A_2, A_3, for every such w, two of these three preferred nonedges do not contain w.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.).

Lemma. *Let $G=(V,E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w\in V$ such that $G-w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A. Then there is a $z' \in A$ that binds a preferred nonedge A'.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A. Then there is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$ and $A \cap A' = \emptyset$.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A. Then there is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$ and $A \cap A' = \emptyset$. Because every $w \in V$ such that $G - w$ is indecomposable is contained in exactly one preferred nonedge, we conclude that $|V| = 4$, a contradiction.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A. Then there is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$ and $A \cap A' = \emptyset$. Because every $w \in V$ such that $G - w$ is indecomposable is contained in exactly one preferred nonedge, we conclude that $|V| = 4$, a contradiction. Hence this last case does not occur, which concludes the proof.

Lemma. *Let $G = (V, E)$ be an indecomposable edge-absent almost-all-cards-decomposable graph. Then there is a $w \in V$ such that $G - w$ is indecomposable and such that there are two distinct vertices z and z' that bind preferred nonedges that do not contain w.*

Proof (cont.). This leaves the case that there are at most 2 distinct preferred nonedges. Let z bind the preferred nonedge A. Then there is a $z' \in A$ that binds a preferred nonedge A'. Moreover, $z \in A'$ and $A \cap A' = \emptyset$. Because every $w \in V$ such that $G - w$ is indecomposable is contained in exactly one preferred nonedge, we conclude that $|V| = 4$, a contradiction. Hence this last case does not occur, which concludes the proof. ∎

Theorem.

Theorem. *Decomposable graphs are set-recognizable.*

Theorem. *Decomposable graphs are set-recognizable.*

Proof.

Theorem. *Decomposable graphs are set-recognizable.*

Proof. Let $G = (V, E)$ be an indecomposable graph.

Theorem. *Decomposable graphs are set-recognizable.*

Proof. Let $G = (V, E)$ be an indecomposable graph. If G is neither all-cards-decomposable, nor almost-all-cards-decomposable, then there is no decomposable graph with the same set of cards as G.

Theorem. *Decomposable graphs are set-recognizable.*

Proof. Let $G = (V, E)$ be an indecomposable graph. If G is neither all-cards-decomposable, nor almost-all-cards-decomposable, then there is no decomposable graph with the same set of cards as G.
If G is all-cards-decomposable, then G is set-reconstructible and hence there is no graph with the same set of cards as G.

Theorem. *Decomposable graphs are set-recognizable.*

Proof. Let $G = (V, E)$ be an indecomposable graph. If G is neither all-cards-decomposable, nor almost-all-cards-decomposable, then there is no decomposable graph with the same set of cards as G.
If G is all-cards-decomposable, then G is set-reconstructible and hence there is no graph with the same set of cards as G.
So let G be an indecomposable edge-absent almost-all-cards-decomposable graph.

Theorem. *Decomposable graphs are set-recognizable.*

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.).

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Every vertex of G binds at most one preferred nonedge.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Every vertex of G binds at most one preferred nonedge. That is, for every decomposable card $G-x$ of G, among the autonomous sets of vertices of $G-x$, there is exactly one autonomous set of vertices that is an independent set of 2 vertices.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Every vertex of G binds at most one preferred nonedge. That is, for every decomposable card $G-x$ of G, among the autonomous sets of vertices of $G-x$, there is exactly one autonomous set of vertices that is an independent set of 2 vertices.

Let $w \in V$ be a vertex that does not bind any subset of V.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Every vertex of G binds at most one preferred nonedge. That is, for every decomposable card $G-x$ of G, among the autonomous sets of vertices of $G-x$, there is exactly one autonomous set of vertices that is an independent set of 2 vertices.
Let $w \in V$ be a vertex that does not bind any subset of V. Then there are distinct $z, z' \in V \setminus \{w\}$ such that each binds a preferred nonedge that does not contain w.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Every vertex of G binds at most one preferred nonedge. That is, for every decomposable card $G-x$ of G, among the autonomous sets of vertices of $G-x$, there is exactly one autonomous set of vertices that is an independent set of 2 vertices.
Let $w \in V$ be a vertex that does not bind any subset of V. Then there are distinct $z, z' \in V \setminus \{w\}$ such that each binds a preferred nonedge that does not contain w. In particular, each of them, binds a preferred nonedge in $G-w$.

Theorem. *Decomposable graphs are set-recognizable.*

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.).

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T = (W, F)$ be the index graph of the canonical decomposition of H.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T=(W,F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G-w$.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T=(W,F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G-w$. By the preceding paragraph, there are $z_T, z'_T \in W$ such that each binds a doubleton of independent vertices in T.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T = (W, F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G - w$. By the preceding paragraph, there are $z_T, z'_T \in W$ such that each binds a doubleton of independent vertices in T. Let z_T not be the index of the autonomous nonedge of H.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T = (W, F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G - w$. By the preceding paragraph, there are $z_T, z'_T \in W$ such that each binds a doubleton of independent vertices in T. Let z_T not be the index of the autonomous nonedge of H. Let $v(z_T)$ be the vertex of H that is indexed by z_T.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T=(W,F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G-w$. By the preceding paragraph, there are $z_T, z'_T \in W$ such that each binds a doubleton of independent vertices in T. Let z_T not be the index of the autonomous nonedge of H. Let $v(z_T)$ be the vertex of H that is indexed by z_T. Then $H-v(z_T)$ contains at least two autonomous doubletons of independent vertices.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T=(W,F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G-w$. By the preceding paragraph, there are $z_T, z'_T \in W$ such that each binds a doubleton of independent vertices in T. Let z_T not be the index of the autonomous nonedge of H. Let $v(z_T)$ be the vertex of H that is indexed by z_T. Then $H-v(z_T)$ contains at least two autonomous doubletons of independent vertices. Because every decomposable card of G contains exactly one autonomous doubleton of independent vertices, this means that G and H cannot have the same set of unlabelled cards.

Theorem. *Decomposable graphs are set-recognizable.*

Proof (cont.). Suppose for a contradiction that there is an edge-absent decomposable graph H with the same set deck as G. Let $T=(W,F)$ be the index graph of the canonical decomposition of H. Then T is isomorphic to $G-w$. By the preceding paragraph, there are $z_T, z_T' \in W$ such that each binds a doubleton of independent vertices in T. Let z_T not be the index of the autonomous nonedge of H. Let $v(z_T)$ be the vertex of H that is indexed by z_T. Then $H-v(z_T)$ contains at least two autonomous doubletons of independent vertices. Because every decomposable card of G contains exactly one autonomous doubleton of independent vertices, this means that G and H cannot have the same set of unlabelled cards. ■